Dynamic models in transportation

André de Palma

Cergy Paris Université

In collaboration with Robin Lindsey (UBC)

March, 10, 2025

Dynamic models

Dynamic models account for changes in traffic flows over time. These models include dynamics on the supply side, dynamics on the demand side, or both.

Supply-side models

Describe how traffic flows evolve over time and space.

Hydrodynamic model (Lighthill & Whitham, 1955; Richards, 1956) Two fluids model Ilya Prigogine and Robert Herman

Extensions MFD (C. Daganzo, 1997, N. Geroliminis)

Demand-side models

Describe travelers' trip-timing preferences.

Combined models

Queuing or bottleneck congestion (Vickrey, 1969; Arnott et al., 1990, 1993; …).

Flow congestion (Henderson, 1974).

References

Need to read:

Arnott, R., A. de Palma and R. Lindsey (1993), A Structural Model of Peak-Period Congestion: A Traffic Bottleneck with Elastic Demand, *American Economic Review*, 83, 161-179.

Tutorial of METROPOLIS (needed for the Lab.)

Look also at (if we wish) some chapters in..

de Palma, A., R. Lindsey, E. Quinet and R. Vickerman (eds.) 2011, *Handbook in Transport Economics*, Volume 1 & 2, Edgar Eldgard, (880p).

The bottleneck model

One O-D pair, one link. (e.g. commuting from a suburb to the CBD.)

N identical individuals travel in identical vehicles.

One decision: departure time.

Supply-side assumptions

Travel is free-flowing (i.e., uncongested) except at a bottleneck.

Normalize free-flow time to zero.

s: bottleneck capacity [cars/hour]

$Q(t)$: queue length upstream of bottleneck [cars]. Vertical queue.

$T(t)$: travel time experienced by commuter departing at time t:

(1) $T(t) = Q(t)/s.$

The bottleneck model

Demand-side assumptions

t^* : preferred arrival time at work.

Travelers incur a *schedule delay cost* if they arrive earlier or later than t^*.

Travel cost function:

(2) $C(t) = \alpha$ (travel time) + β (time early) + γ (time late).

Require $\beta < \alpha$ (true empirically)

$\tau(t)$: toll paid at time t

$p(t)$: full price or generalized cost for departure at time t

$$p(t) = C(t) + \tau(t)$$

The bottleneck model: No-toll equilibrium

t_0 : beginning of departure period.

t_e : end of departure period.

Since the bottleneck operates at capacity during (t_0, t_e),

(3) $t_e - t_0 = N / s.$

Equilibrium cost for first driver:

(4a) $p(t_0) = C(t_0) = \beta(t^* - t_0).$

Equilibrium cost for last driver:

(4b) $p(t_e) = C(t_e) = \gamma(t_e - t^*)$

The last driver incurs no queuing delay. **Q: Why?**

The bottleneck model: No-toll equilibrium

Equal-trip-price condition:

(4c) $$p\left(t_0\right) = p\left(t_e\right) \equiv p.$$

Solution (3, 4a, 4b, 4c):

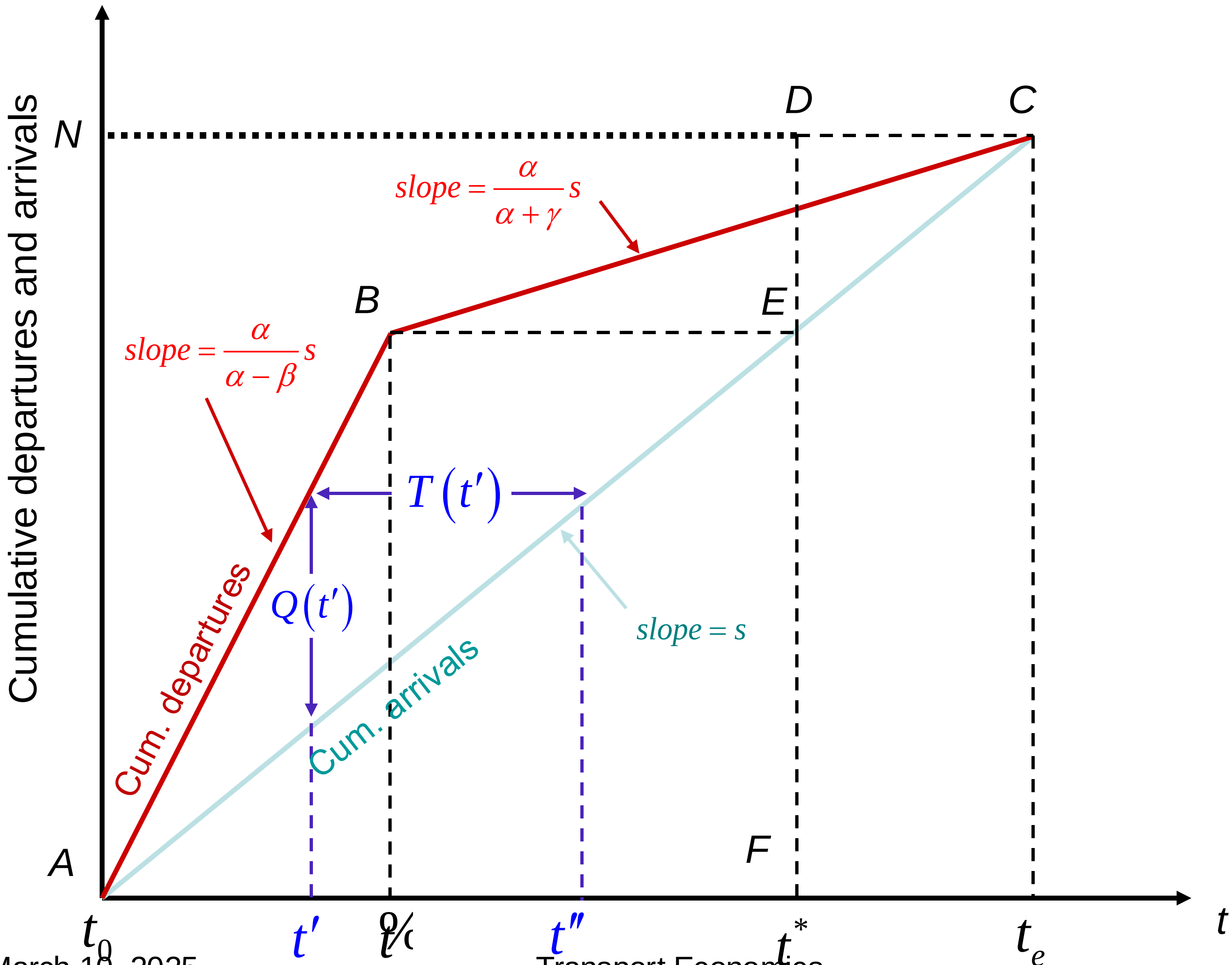
(5a,b) $$t_0 = t^* - \frac{\gamma}{\beta+\gamma}\frac{N}{s}, \quad t_e = t^* + \frac{\beta}{\beta+\gamma}\frac{N}{s}.$$

(6) $$p^n = C^n = \frac{\beta\gamma}{\beta+\gamma}\frac{N}{s}$$

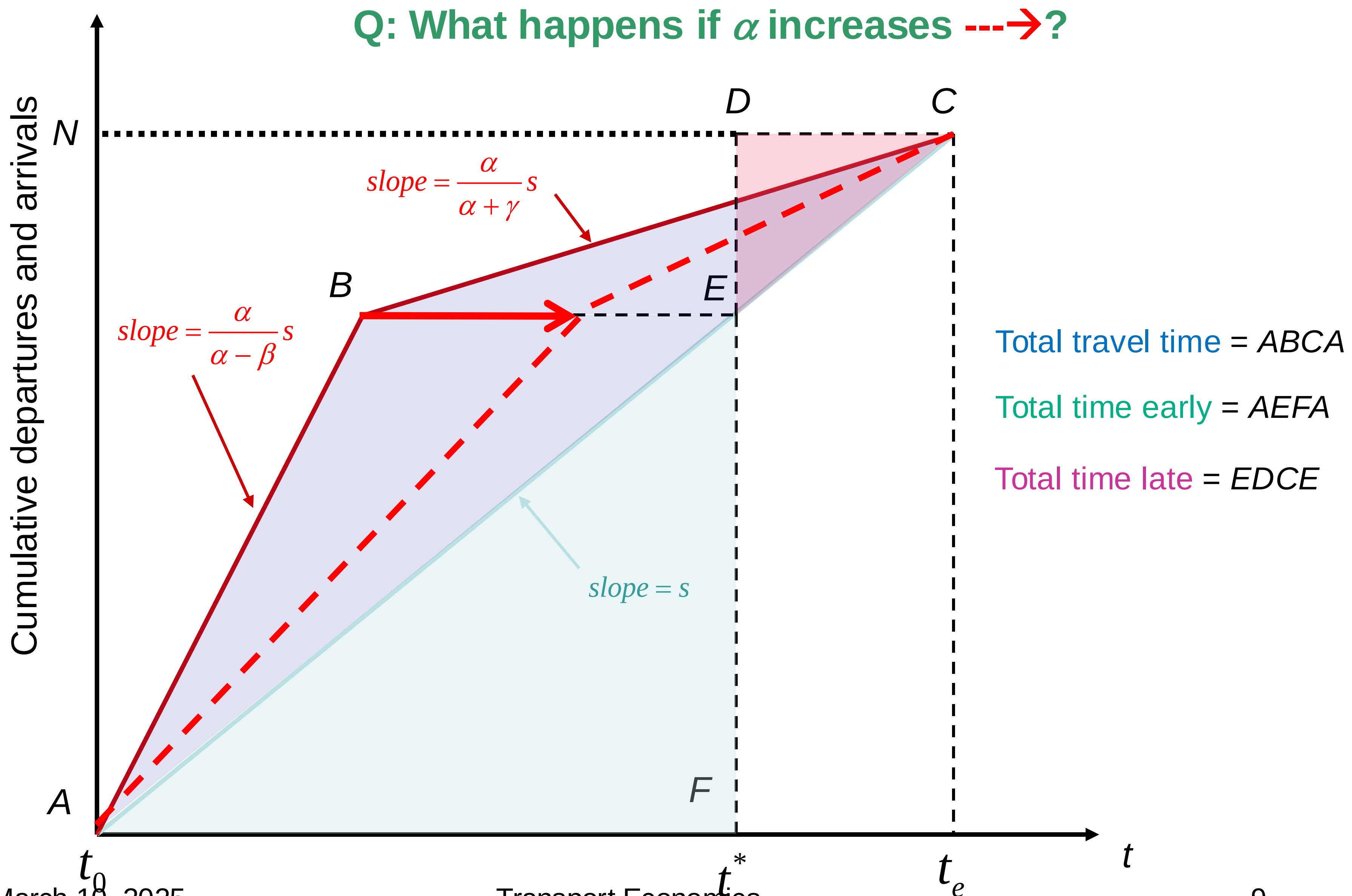
t_0, t_e and p are all independent of α. **Q: Why?**

Exercise: compute the evolution of travel time over time @4:00

The bottleneck model: No-toll equilibrium



The bottleneck model: No-toll equilibrium



The bottleneck model: No-toll equilibrium

Recall $C^n = \frac{\beta\gamma}{\beta+\gamma}\frac{N}{s}$ Define $\delta \equiv \frac{\beta\gamma}{\beta+\gamma}$ $C^n = \delta\frac{N}{s}$

(7a) $TC^n = \delta\frac{N^2}{s}.$

(7b) $MC^n = \frac{\partial TC^n}{\partial N} = 2\delta\frac{N}{s}.$

(7c) $MEC^n = MC^n - C^n = \delta\frac{N}{s}$ $\left(= C^n\right)$

Total travel time cost $TTC^n = \frac{\delta}{2}\frac{N^2}{s}.$

Total schedule delay cost $SDC^n = \frac{\delta}{2}\frac{N^2}{s}$

(Equality of TTC^n and SDC^n can be confirmed by comparing areas in the preceding figure.)

The bottleneck model: Social optimum

1. Optimal to use bottleneck at full capacity: $t_e - t_0 = N / s$.
2. To minimize total schedule delay costs, the first and last persons incur equal costs.
3. Since queuing is wasteful, hold departure rate = capacity.

Implications:

- Departure and arrival times are the same as for no-toll equilibrium
- Travel delay costs = 0. Congested travel time equals toll revenue
- Optimum Total travel cost = Total schedule delay cost:

(8)

$$TC^0 = \frac{\delta}{2}\frac{N^2}{s} \quad \left(= \frac{1}{2} TC^n \right) \qquad AC^0 = \frac{\delta}{2}\frac{N}{s} \quad \left(= \frac{1}{2} AC^n \right).$$

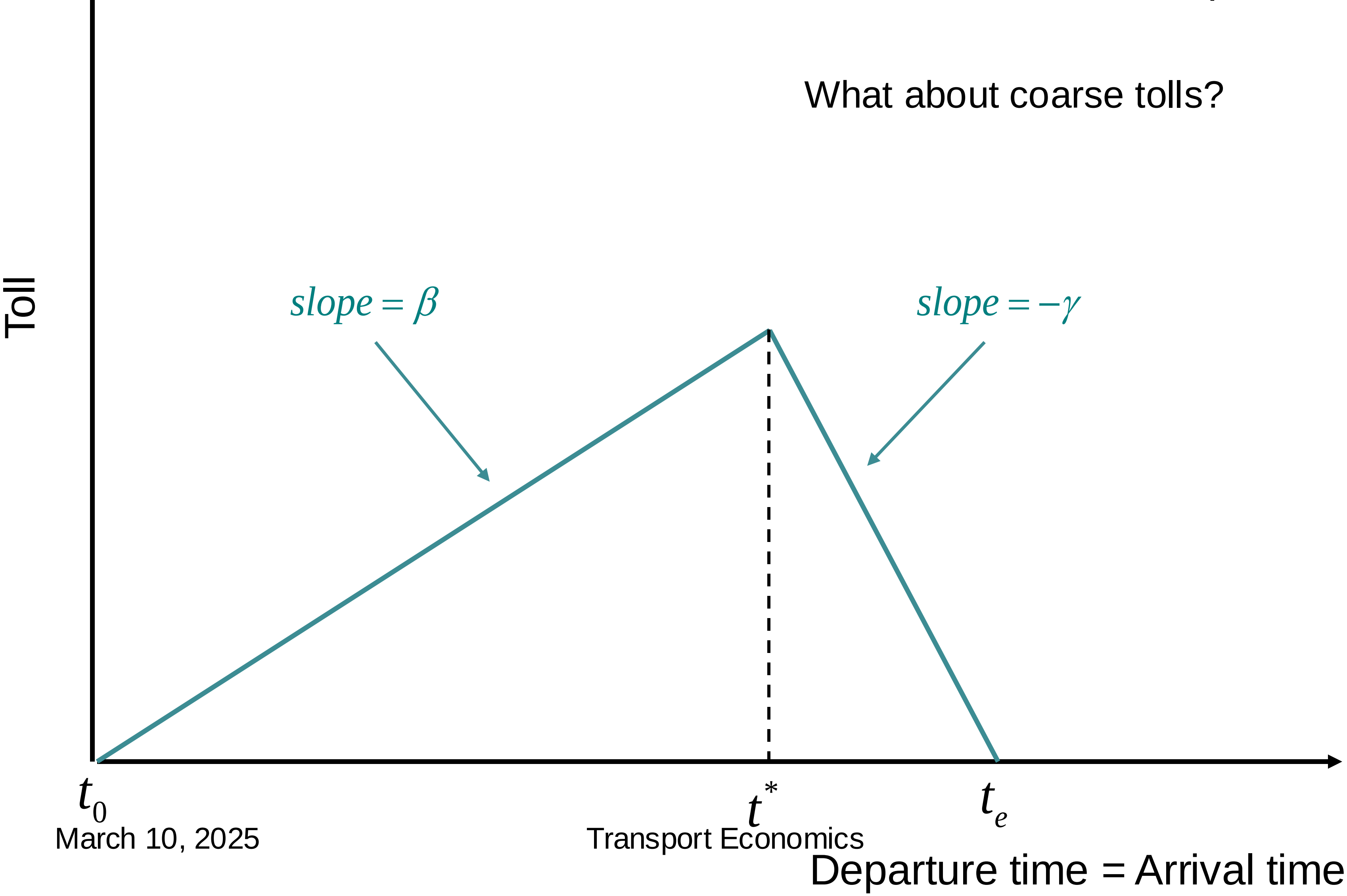
(9)

$$MC^o = \frac{\partial TC^o}{\partial N} = \delta \frac{N}{s}. \quad \left(= \frac{1}{2} MC^n \right)$$

The bottleneck model: Social optimum

Fine toll set to replace queuing time as rationing mechanism for desirable arrival times. Toll revenue = travel time cost at equilibrium.

What about coarse tolls?



Optimal capacity and self financing

$$W = B - \delta \frac{N^2}{2s} - \xi s; \ \xi s = \text{ construction cost.}$$

$$\frac{dW}{ds} = \delta \frac{N^2}{2s^2} - \xi = 0 \ \text{ or } \ s^\circ = N\sqrt{\frac{\delta}{2\xi}}$$

Toll revenue (=Congested equil. travel time):

$$\frac{\delta}{2}\frac{N^2}{s^0} = \delta \frac{N^2}{2N\sqrt{\frac{\delta}{2\xi}}} = N\sqrt{\frac{\delta\xi}{2}}$$

Construction cost: $\xi s^\circ = \xi N\sqrt{\frac{\delta}{2\xi}} = N\sqrt{\frac{\delta\xi}{2}}$ The same!

Maintenance: discussion

- When to maintain road
- Why often roods and bridge at not well enough maintained?
- What to do about that?

Road Damage Externalities and Road User Charges

David M. Newbery

Econometrica, Volume 56, Issue 2 (Mar., 1988), 295-316.

The bottleneck model: User heterogeneity

Heterogeneity in vehicle characteristics

- Size, acceleration rate, visual intrusion, axle weight, etc.
- Typically measured using Passenger Car Equivalents (PCEs).
- If a truck has a PCE=2, the optimal truck toll is twice the car toll.

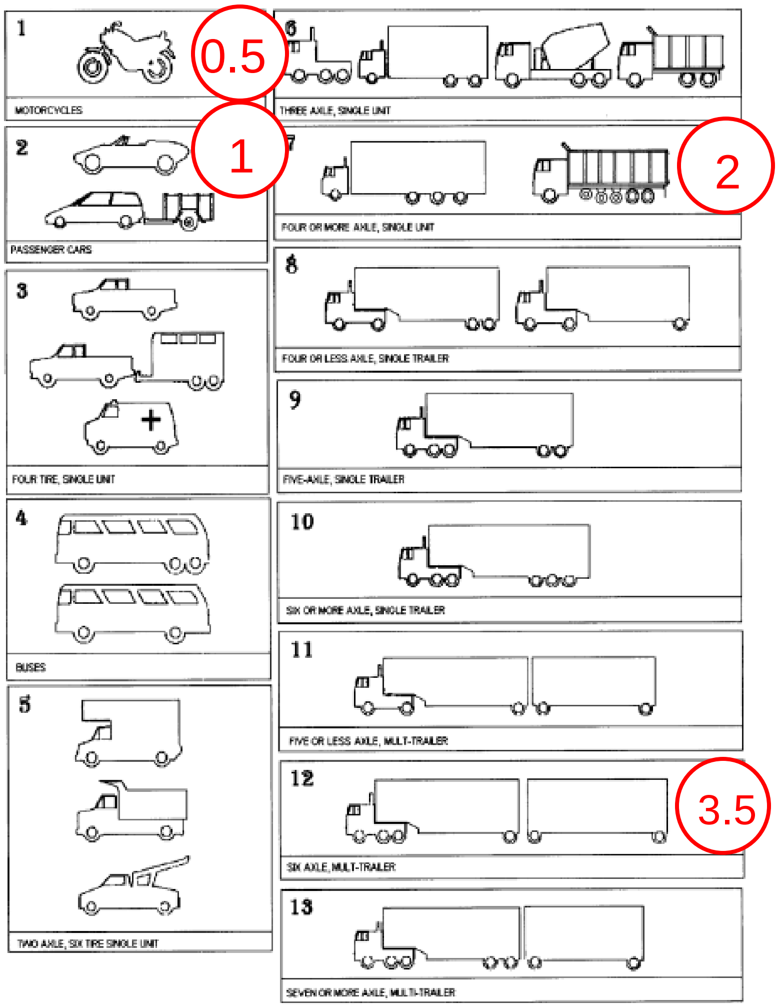


FIGURE 3.1. FHWA Classification of vehicles. (Source: FHWA Traffic Monitoring Guide)

Toll differentiation is feasible since vehicle characteristics visible.

The bottleneck model: User heterogeneity

Heterogeneity in driver characteristics

Preference parameters:

- Willingness to pay for trip, value of travel time, distance
- In the bottleneck model: α, β, γ and t^*.

Q: When does driver heterogeneity call for toll differentiation?

A: Only if it affects drivers' decisions. Otherwise, tolls can be anonymous.

The following two drivers behave in the same way:

Driver 1: α, β, γ, t^*

Driver 2: 2α, 2β, 2γ, t^*

Local and global pollution:

Tolls

Local and global pollution:

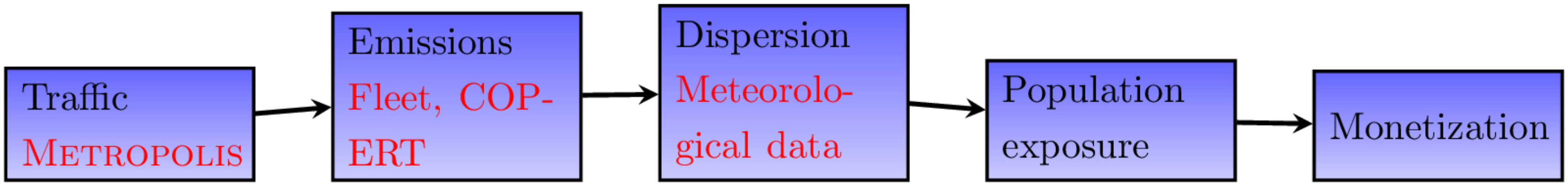
Vosough S, A. de Palma & R. Lindsey (2022). Pricing vehicle emissions and congestion externalities using a dynamic traffic network simulator, *Transportation Research* A, 161, 1-24.

Assessment

Le Frioux, R., A. de Palma & N. Blond (2023). Assessing the Economic Costs of Road Traffic-Related Air Pollution in La Réunion.

https://thema.u-cergy.fr/IMG/pdf/2023-09.pdf

Integrated chain of model, METRO-TRACE
(Traffic related Air Pollution Cost Evaluation).



- **Traffic Model:** This model generates vehicle movements of various types
- **Emission Model**: Using the outputs of the traffic model as input variables, this model calculates the emissions of different local and global pollutants.
- **Pollutant Dispersion Model**: Focused on local pollutants, this model uses Gaussian dispersion techniques. Parameters: temperature and wind speed.
- **Exposure Model**: This model determines pollutant exposure levels for different agents at any given moment, based on their locations. Since the transport model provides the precise location of each agent throughout the day
- **Monetization of Health Impacts**: The health impacts of pollutant exposure are monetized using epidemiological data available in medical literature.

Speed reduction

Reducing speed to 70 $km.h^{-1}$ for speed lower than 90 $km.h^{-1}$		
	Value (k€)	**Diff.**
Travel time costs	41,507	-104
Fuel consumption costs	4,204	-25
Environmental costs	567	-4
Population exposure costs	952	-4
Total	47,230	-137
Reducing speed by 20 $km.h^{-1}$ for speed from 90 to 110 $km.h^{-1}$		
	Value (k€)	**Diff.**
Travel time costs	42,100	+489
Fuel consumption costs	4,243	+14
Environmental costs	572	+1
Population exposure costs	948	-8
Total	47,863	+496
Reducing speed by 20 $km.h^{-1}$ for speed from 110 to 130 $km.h^{-1}$		
	Value (k€)	**Diff.**
Travel time costs	41,451	-160
Fuel consumption costs	4,213	-16
Environmental costs	958	+3
Population exposure costs	380	+2
Total	47,002	-177

Videos on Climate Change

Action, Inaction, and Climate Change

https://youtube.com/@AICC-Academia